Midterm (20 points)
Multiple Choice: Select the best answer for each of the following questions. Put a circle around your final answer.

(2pts) **1.** In units of volts, the electric potential in a region of space is given by $V = 4x^2 - 6y^3z$. In this expression, x and y represent coordinates in space and are measured in units of meters. The corresponding electric field in this region of space, in units of V/m, is:

(a) $\frac{4}{3}x^3\,\hat{\imath} - \frac{3}{2}y^4z\,\hat{\jmath} - 3y^3z^2\,\hat{k}$

(b) $-\frac{4}{3}x^3\,\hat{\imath} + \frac{3}{2}y^4z\,\hat{\jmath} + 3y^3z^2\,\hat{k}$

(c) $8x\,\hat{\imath} - 18y^2z\,\hat{\jmath} - 6y^3\,\hat{k}$

(d) $-8x\,\hat{\imath} + 18y^2z\,\hat{\jmath} + 6y^3\,\hat{k}$

(e) $6z\,\hat{k}$

(f) The electric field is zero.

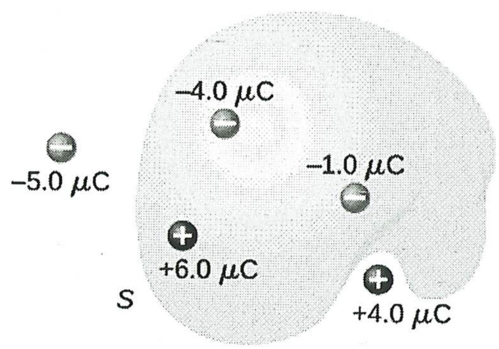
$E_s = -\frac{dV}{ds}$

$E_x = -\frac{dV}{dx} = -8x \quad E_y = -\frac{dV}{dy} = 18y^2z$

$E_z = -\frac{dV}{dz} = 6y^3$

$\therefore \vec{E} = -8x\,\hat{\imath} + 18y^2z\,\hat{\jmath} + 6y^3\,\hat{k}$

(2pts) **2.** What is the net electric flux through the closed surface S shown in the figure below? Note that $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2N^{-1}m^{-2}}$.



$\Phi = \frac{q_{encl}}{\varepsilon_0} = \frac{(-4\mu C - 1\mu C + 6\mu C)}{\varepsilon_0}$

$= \frac{+1\mu C}{\varepsilon_0} = 1.13 \times 10^5\ \frac{Nm^2}{C}$

(a) $1.1 \times 10^{-10}\ \mathrm{Nm^2/C}$

(b) $-1.1 \times 10^{-10}\ \mathrm{Nm^2/C}$

(c) $1.1 \times 10^{5}\ \mathrm{Nm^2/C}$

(d) $-1.1 \times 10^{5}\ \mathrm{Nm^2/C}$

(e) $1.1 \times 10^{11}\ \mathrm{Nm^2/C}$

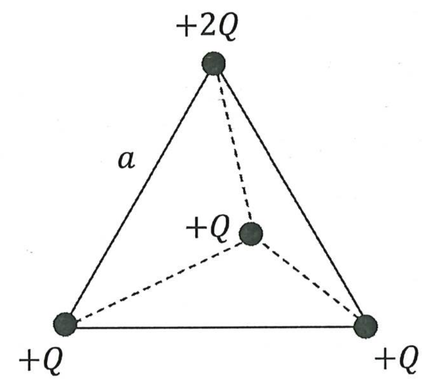
(f) $-1.1 \times 10^{11}\ \mathrm{Nm^2/C}$

(g) The net electric flux through the sphere is zero.

4 pts

Free Response: Write out complete answers to the following questions. Include diagrams where appropriate. Show your work since it allows us to award partial credit.

(4pts) **3.** A tetrahedron is a three-sided pyramid. Consider the collection of charges fixed at the corners of a symmetric tetrahedron as shown in the figure below. The distance from one charge to any other charge is $a = 3.0$ mm.



6 pairs of charges

The three $+Q$ charges at the base are permanently fixed in place. If the $+2Q$ charge is released from rest from the position shown in the figure, how fast is it moving once it is infinitely far away from the $+Q$ charges? Assume $Q = 1.0 \times 10^{-9}$ C and that the mass of the $+2Q$ charge is 5.0 mg. (**4 marks**)

initial: $K_i = 0 \quad U_i = 3\frac{k_e(2Q \cdot Q)}{a} + 3\frac{k_e(Q \cdot Q)}{a}$

$= \frac{6k_eQ^2}{a} + \frac{3k_eQ^2}{a}$

final: $K_f = \frac{1}{2}mv^2 \quad U_f = \frac{3k_eQ^2}{a}$ (just 3 charges at the base)

conservation of energy:

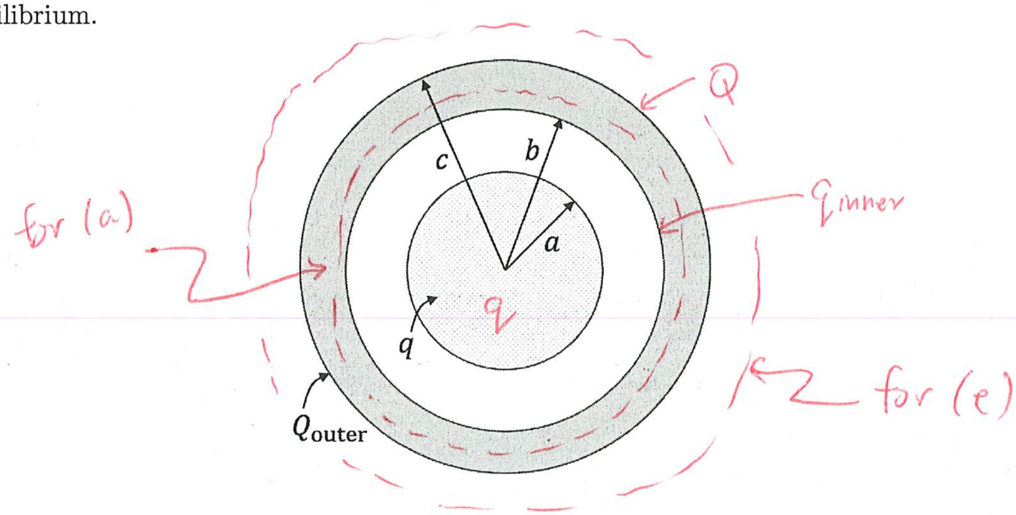
$\therefore K_i + U_i = K_f + U_f$

$\frac{6k_eQ^2}{a} + \frac{3k_eQ^2}{a} = \frac{1}{2}mv^2 + \frac{3k_eQ^2}{a}$

$\therefore v = \sqrt{\frac{12k_eQ^2}{ma}} = \boxed{2.68\ m/s}$

4 pts

(7pts) **4.** Consider a *conducting* spherical shell with inner radius b and outer radius c. A *conducting* sphere of radius a and charge $q = 2.5\ \mu\text{C}$ is placed at the centre of the cavity. The *outer* surface of the shell has charge $Q = 4.0\ \mu\text{C}$. Assume that the conductors are in electrostatic equilibrium.



(a) What is the charge on the *inner* surface of the conducting shell? (**2 marks**)

$\vec{E} = 0$ inside conductor $\quad \oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0} = \frac{(q + q_{inner})}{\varepsilon_0} = 0 \quad \therefore q_{inner} = -q = -2.5\,\mu C$

(b) What is the *net* charge on the conducting shell? (**1 mark**)

$q_{shell} = q_{inner} + Q_{outer} = -q + Q = 1.5\,\mu C$

(c) What is the magnitude of the electric field a distance $r = 1.0$ cm away from the centres of the concentric spheres assuming that $r < a$? (**1 mark**)

$r < a$ ∴ inside conducting sphere $\boxed{\vec{E} = 0}$

(d) What is the magnitude of the electric field a distance $r = 4.0$ cm away from the centres of the concentric spheres assuming that $b < r < c$? (**1 mark**)

$b < r < c$ is inside conducting shell $\boxed{\vec{E} = 0}$

(e) What is the magnitude of the electric field a distance $r = 8.0$ cm away from the centres of the concentric spheres assuming that $r > c$? (**2 marks**)

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$

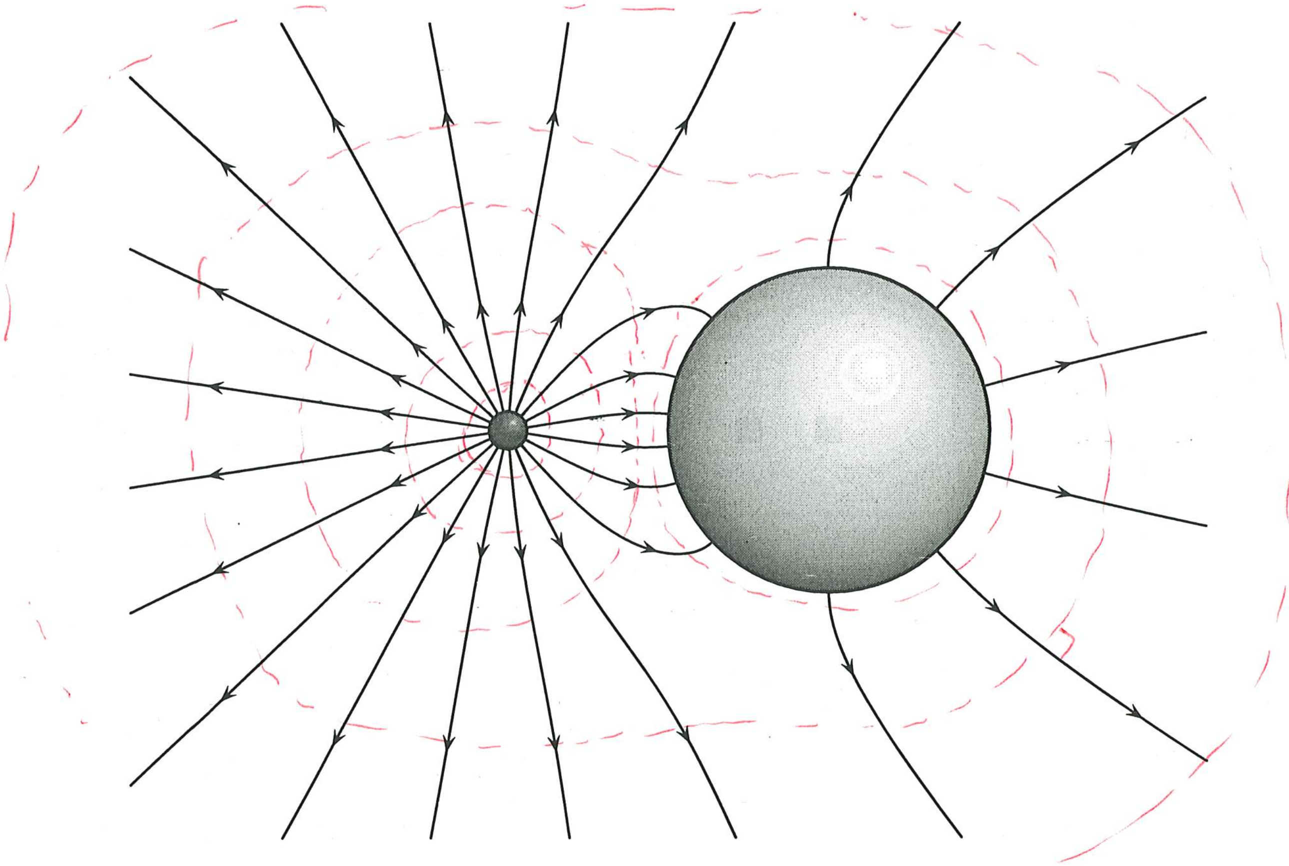
⇓

$E\,4\pi r^2 = \frac{(Q_{outer} + q_{inner} + q)}{\varepsilon_0}$

$\therefore E = \frac{(4.0 - 2.5 + 2.5)\,\mu C}{4\pi r^2 \varepsilon_0} = \frac{4.0\,\mu C}{4\pi\varepsilon_0 (.08\,m)^2} = \boxed{5.63\times10^6\ \frac{N}{C}}$

7 pts

(5pts) **5.** The figure below shows a cross-section of a neutral spherical conductor (represented by the large circle) immersed in the electric field due to a point charge (represented by the small circle). The lines are electric field lines.



(a) Add dashed lines to the figure to represent lines of equipotential. The distance between each pair of equipotentials should represent approximately the same change in electrical potential ΔV. Draw your dashed lines as accurately as possible. Include lines that are near the two spheres and also lines that are far from the two spheres. (**3 marks**)

equipotentials ⊥ $\vec{E}$-field.

(b) Assuming that the conductor is in electrostatic equilibrium, is charge at the left side of the sphere greater than, less than, or equal to the charge at the right side of the sphere? Explain your reasoning. Writing down only the correct answer will only earn half of the possible marks. (**1 mark**)

$\vec{E}$ pt. away from pt. charge ∴ it is positive.

Left side is negative. Right side is positive

$\vec{E}$-field lines pt away from positive charges (right side of conductor)

$\vec{E}$-field lines point towards neg. charges (left side of conductor)

(c) Assuming that the conductor is in electrostatic equilibrium, is potential at the left side of the sphere greater than, less than, or equal to the potential at the right side of the sphere? Explain your reasoning. Writing down only the correct answer will only earn half of the possible marks. (**1 mark**)

$\Delta V = \int \vec{E} \cdot d\vec{s}$ since $\vec{E} = 0$ inside conductor in equilibrium, $\Delta V = 0$.

∴ Entire conductor is at a constant potential. $V_{left} = V_{right}$.

5 pts